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Fractional Quantum Point Contact Conductance Quantization Produced by an Electric Field

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Abstract

It is shown that the zero-temperature linear-response quantum point contact (QPC) conductance is fractionally quantized in units $2e^2/h$ whenever the zero-field through QPC transmission coefficient is quantized. The same origins has the predicted effect of splitting in the conductance spectrum of sharp transmission peaks and dips. For a QPC modeled by a 2D rectangular channel, an analytical expression for the differential nonlinear conductance is proposed which includes effects of an abrupt potential drop at the QPC borders with the electron gas reservoirs and a linear potential variation inside the channel.

INTRODUCTION

Advances in nanostructure technology made available small structures (devices) which can be passed through by electrons stic regime of electron motion are exhibited by a quantum point contact (QPC) [1]. (Here, the term QPC refers to constrictions of arbitrary geometry in a two-dimensional electron gas (2DEG) with a characteristic width of the order of the Fermi wavelength). One of the first quantum effects discovered in QPCs was the conductance quantization [2,3]. An elementary explanation of the phenomenon relies on the conductance quantization in an ideal infinite 2D wire which is due to the fact that each propagating electron state associated with subbands (modes) of the transverse quantization contributes $2e^2/h$ to the wire conductance. Naturally, the manifestation of this effect in a real QPC depends on how much the latter looks like the ideal wire.

Despite the origins of the QPC conductance quantization are in general well understood, there are some contradictions in theoretical predictions with regard to the observation of the effect. Glazman and Khaetskii [4] showed that in addition to integer steps in the QPC conductance dependence on the Fermi energy, also half integer steps should appear in the same dependence, if the applied voltage V is dropped equally on both sides of an adiabatic contact. In subsequent papers of Martin-Moreno *et. al.* [5] and Hongqi Xu [6], the prediction of the half quantization of the conductance was confirmed for QPCs modeled by a saddle-point potential and a rectangular channel, respectively. However, the exact quantum mechanical calculations of Castaño and Kirchenow [7] and Lent, Sivaprakasam, and Kirkner [8] performed for the case of a linear variation of an electric potential along a rectangular channel are in conflict with the prediction of Glazman and Khaetskii – no structure that could be interpreted as an additional quantization has been found in the conductance-vs-Fermi energy dependence. At the same time, as distinct from the very first experiments of van Wees *et. al.* [2] and Wharam *et. al.* [3], the half quantization was really observed by Patel *et. al.* [9].

The aim of this work is to show that the effect of the fractional quantization can be expected whenever the applied voltage V shifts the opening energies of the propagating modes in the contact. Besides, the QPC conductance for a rectangular channel is presented in an analytical form which accurately describes nonlinear electric field effects. To our knowledge, the corresponding expression for the transmission coefficient (equation (6)) is found for the first time.

GENERAL CONSIDERATION

The linear-response zero-temperature QPC conductance g_0 (in units $2e^2/h$) is often identified with the through QPC transmission coefficient T_0 taken at the Fermi energy E_F^0 [1]

$$g_0 = T(\varepsilon = E_F^0, V = 0) \equiv T_0(E_F^0), \tag{1}$$

though a precise form of the Landauer formula valid at zero temperature and arbitrary electric potentials reads

$$g = \frac{\partial}{\partial eV} \int_{E_F^0 - eV}^{E_F^0} T(\varepsilon, V) d\varepsilon. \tag{2}$$

Consider a symmetric QPC. In this case, the applied voltage shifts the Fermi energy in 2DEG on both sides of the contact equally but with opposite signs of the shift in the source and drain reservoirs, see Fig. 1a. One may assume therefore that $T(\varepsilon_V, V) = T(\varepsilon_V, -V)$, where $\varepsilon_V = \varepsilon + eV/2$ ($T(\varepsilon_V)$ and $T(\varepsilon)$ are of course different functions but we preserve for them the same notation). For the model in focus, equation (2) can be rewritten in the linear approximation as

$$g_0 = \frac{\partial}{\partial eV} \int_{E_F^0 - eV/2}^{E_F^0 + eV/2} T_0(\varepsilon_V) d\varepsilon_V = \frac{1}{2} \left(T_0(E_F^0 + eV/2) + T_0(E_F^0 - eV/2) \right). \tag{3}$$

Evidently that in comparison with definition (1), equation (3) predicts a very much different behavior of g_0 in the energy interval eV, whenever the zero field transmission coefficient T_0 as a function of energy possesses a singular behavior in the same scale. In particular, (3) predicts that the QPC conductance in units $2e^2/h$ is quantized as half integers, if the transmission coefficient is quantized as integers.

At the same time, it is easy to show that equation (1) is valid in linear response, if the Fermi energy in the source 2DEG remains unshifted by an applied voltage as it was assumed by Castaño and Kirczenow [7]. Therefore, it is not at all surprising that the fractional conductance quantization does not reveal itself in calculations performed for a QPC model used in [7]. It is also interesting to note that a QPC used in experiments of van Wees *et. al.* [2] was not symmetric (no fractional quantization observed), whereas that one used by Patel *et. al.* [9] was symmetric (experimental confirmation of fractional quantization).

It is worth emphasizing that in accordance with (3) any singular structure in the zero field transmission spectrum (such as sharp resonance peaks and dips) will be splitted in the conductance spectrum of a symmetric QPC.

RECTANGULAR CHANNEL

Consider now the fractional conductance quantization effect in a QPC shaped as a rectangular channel.

In accordance with the Landauer approach [10], to calculate the QPC conductance one has to know the energy dependence of the through QPC total transmission coefficient

$$T(\varepsilon_V, V) = \sum_{j_s=1}^{\bar{j}_s} \sum_{j_d=1}^{\bar{j}_d} |t_{j_d j_s}|^2, \tag{4}$$

where $t_{j_d j_s}$ is the probability amplitude of the transmission from j_s mode in the source reservoir to j_d mode of the drain reservoir, and $\bar{j}_s$ and $\bar{j}_d$ are the highest propagating modes in the source

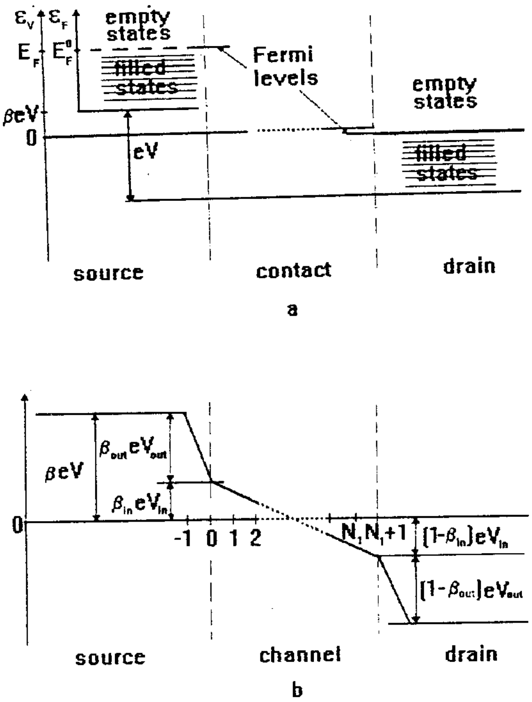

Figure 1: Occupation of electron states in the source and drain reservoirs connected by a QPC at zero temperature – a, and the electric potential drop distribution accepted for the given QPC model – b. The total voltage drop $V = V_{out} + V_{in}$ is step-like at the QPC entrance and exit and varies linearly inside the QPC. The particular case $V_{out} = 0$, $\beta_{in} = 0$ corresponds to the QPC model used in [7,8], and $V_{in} = 0$ – in [6]; the case $\beta_{out} = \beta_{in} = \beta = 1/2$ represents a symmetric QPC. The notation $\varepsilon = \varepsilon_V - \beta eV$ ($E_F^0 = E_F - \beta eV$), see Eqs. 1 and 2, denotes an electron energy (the Fermi energy) counted from the conductance band bottom in the source reservoir.

and drain reservoirs, respectively. The transmission and reflection probability amplitudes are related to the amplitude of an incident flux by a set of linear equations which can be easily found from the condition of the continuity of the electron wave function and of its derivatives [7]. An alternative approach exploits the Green-function method in the tight binding formulation of the corresponding scattering problem [11-13]. In the latter case, a discrete lattice in the form of a wide-narrow-wide structure is used to mimick the QPC geometry. The dynamics of a free electron in the lattice is determined by the electron transfer energy L between the nearest lattice sites. The electron energy is confined within the interval $4|L|$ and at any sites, except those lying at the borders, can be represented in units $2|L|$ by

$$\varepsilon_V = \begin{cases} 2 - \cos k_{j_s} - \cos \frac{\pi j}{\mathcal{N}+1} - \beta eV/2, & \text{in source,} \\ 2 - \cos k_j - \cos \frac{\pi j}{N+1} + \left(\frac{n-1}{N_1-1} - \beta_{in} eV_{in}\right)(1 - \delta_{N_1 1}), & \text{in channel,} \\ 2 - \cos k_{j_d} - \cos \frac{\pi j}{\mathcal{N}+1} + (1-\beta)eV/2, & \text{in drain.} \end{cases} \quad (5)$$

In (5), $\mathcal{N}$ and N are the width of the wide and narrow parts of the wide-narrow-wide structure, respectively, k_j is a dimensionless wave vector of a jth mode plane wave.

Using the technique proposed in [12,13] (see also [14] for more details of the tight-binding QPC model description), one can pass in (4) to new unknowns which in essence represent the wave function amplitudes at the channel border with 2DEG. A remarkable advantage of using the new quantities is that they obey equations which mostly account for the mode mixing effects in the through QPC transmission process in the diagonal approximation. Note that a similar (but not equivalent) approximation, called the mean field approximation, has been proposed by Szafer and Stone [15] in their study of the linear response QPC conductance. In the diagonal approximation, the total transmission coefficient can be found analytically. Omitting the details of the derivation procedure, we present here only the final expression of $T(\varepsilon_V, V)$ found for the electron potential profile shown in Fig. 1b

$$T(\varepsilon_V, V) = 4\sum_{j=1}^{N} \frac{\Re(A_j^+)\Re(A_j^-)\left(\tilde{G}^j_{N_1,1}\right)^2}{\left|\left(\tilde{G}^j_{1,1} - \varepsilon^j_{+V} - iA_j^+\right)\left(\tilde{G}^j_{N_1,N_1} - \varepsilon^j_{-V} - iA_j^-\right) - \left[\tilde{G}^j_{N_1,1}\right]^2\right|^2}, \tag{6}$$

where N and N_1 stand for the channel width and length in the number of lattice sites, respectively, $\varepsilon^j_{\pm V} = 2 - \cos k_{th}j - \varepsilon_{\pm V}$, $k_{th} = \pi/(N+1)$, $\varepsilon_{+V} \equiv \varepsilon + (1-\beta)eV$, $\varepsilon_{-V} \equiv \varepsilon - \beta eV$ $(0 \le \beta \le 1)$, $A_j^\pm = \Re(A_j^\pm) + i\Im(A_j^\pm) \equiv \Re(A_j(\varepsilon_{\pm V})) + i\Im(A_j(\varepsilon_{\pm V}))$,

$$\Re(A_j(\varepsilon)) = \frac{2}{\pi^2}k_{th}\sin^2 k_{th}j \int_0^{\arccos(1-\varepsilon)} dx \frac{\sqrt{1-(2-\cos x-\varepsilon)^2}}{(\cos x - \cos k_{th}j)^2} \begin{cases} \sin^2\left(\frac{\pi x}{2k_{th}}\right) & even\ j \\ \cos^2\left(\frac{\pi x}{2k_{th}}\right) & odd\ j \end{cases}, \tag{7}$$

$$\Im(A_j(\varepsilon)) = \frac{2}{\pi^2}k_{th}\sin^2 k_{th}j \int_{\arccos(1-\varepsilon)}^{\pi} dx \frac{\sqrt{(2-\cos x-\varepsilon)^2-1}}{(\cos x - \cos k_{th}j)^2} \begin{cases} \sin^2\left(\frac{\pi x}{2k_{th}}\right) & even\ j \\ \cos^2\left(\frac{\pi x}{2k_{th}}\right) & odd\ j \end{cases}, \tag{8}$$

and

$$\left[J_{\nu_j}(z)Y_{\nu_j+1-N_1}(z) - Y_{\nu_j}(z)J_{\nu_j+1-N_1}(z)\right]\tilde{G}^j_{n,n'}(\varepsilon_V, V) = \begin{cases} J_{\nu_j}(z)Y_{\nu_j-N_1}(z) - Y_{\nu_j}(z)J_{\nu_j-N_1}(z), & n = n' = 1, \\ J_{\nu_j+1}(z)Y_{\nu_j+1-N_1}(z) - Y_{\nu_j+1}(z)J_{\nu_j+1-N_1}(z), & n = n' = N_1, \\ 2/(\pi z), & n = 1(N_1),\ n' = N_1(1), \end{cases} \tag{9}$$

where $z = (N_1-1)/eV$, $\nu_j = z\varepsilon^j_{+V}$, and $J_\nu(z)$, $Y_\nu(z)$ are the Bessel functions of the first and second order, respectively.

Calculations of the differential conductance using (6) in (2) restore the exact calculations, in particular those ones presented in [6,7,15], with a surprising accuracy.

As was mentioned above, the transmission coefficient defined in (6) includes some QPC models studied previously. In particular, Szafer and Stone [15] found an analytical expression

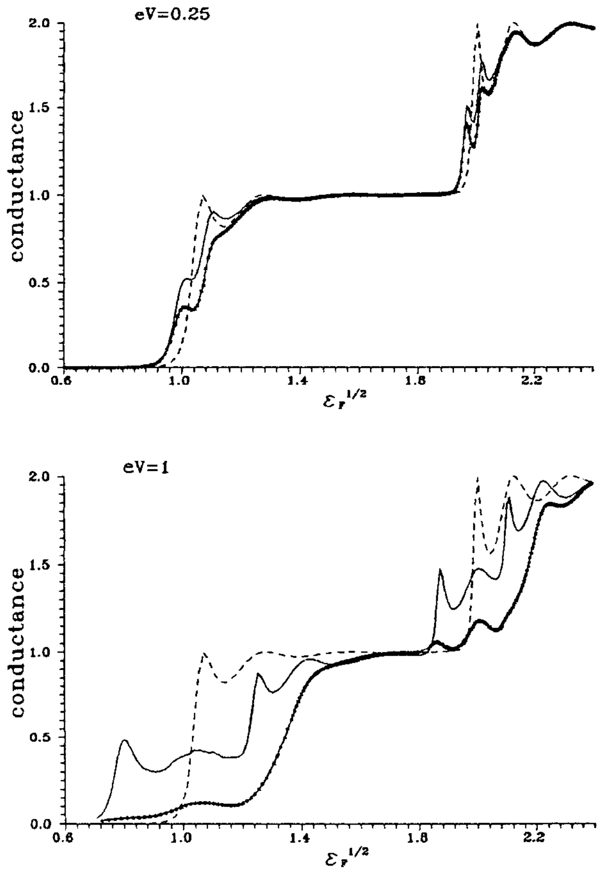

Figure 2: Manifestations of the fractional conductance quantization produced by an electric field in symmetric QPCs: $\beta = 1/2$; $V_{in} = 0$ — solid lines, $V_{out} = 0$ — dotted lines, $V = 0$ — dashed lines. Indicated values of the electric potential difference are given in units of the through channel propagation threshold energy.

for T_0 in the effective mass approximation. Their result generalized with an account to a step-like voltage drop at the channel entrance and exit follows from (6) at $V_{in} = 0$ in the continuum limit, $k_{th} \to 0$, $Na = const$

$$\lim_{k_{th}\to 0} T(\varepsilon_V, V) = T^c(\varepsilon_F, V) =$$

$$= \sum_{j=1}^{\infty} \frac{4q_j^2 \Re(A_j^{c+}) \Re(A_j^{c-}) / \sin^2(\pi q_j l/w)}{\left|\left(q_j \cot(\pi q_j l/w) - iA_j^{c-}\right)\left(q_j \cot(\pi q_j l/w) - iA_j^{c+}\right) - q_j^2 / \sin^2(\pi q_j l/w)\right|^2}, \tag{10}$$

where $\varepsilon_F = q_j^2 + j^2$ denotes the dimensionless Fermi energy in units of the through QPC propagation-threshold energy, and the matrix elements $A_j^{c\pm}$ appeared in (10) are determined by $A_j^{c\pm} = \lim_{k_{th}\to 0} (k_{th})^{-1} A_j^{\pm}$.

At $V = 0$ the expression under the sum sign in (10) differs from that one obtained in Ref. [15] only by the definition of the matrix elements $A_j^{c+} = A_j^{c-}$.

Setting V_{out} in (6) equal to zero, we come to the model of a biased channel without additional potential drops at the channel edges. At $\beta = 0$, the dependence $g(\varepsilon_F)$ coincides perfectly with that calculated by Castaño and Kirczenow [7]. There is no structure which could be interpreted as the half quantization. However, when we set in (6) $\beta \neq 0$, the additional quantization appears, see Fig. 2. For the applied potential much less than the propagation threshold energy, the difference (only quantitative) between the two models, namely $V_{in} = 0$, $V_{out} \neq 0$ and $V_{in} \neq 0$, $V_{out} = 0$, is very small. On the other hand, manifestation of nonlinear field effects are far from being the same in the cases of the abrupt and linear changes in the applied potential. A detailed discussion of these effectes will be published elsewhere.

In summary, it is shown that the fractional quantization is a universal feature of phase-coherent ballistic conduction through a QPC. Its manifestation, however, is strongly dependent on the QPC structure.

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